

Aufgaben zum Lösen von Bruchgleichungen

1.0 Bestimmen Sie die Definitions- und die Lösungsmenge.

1.1 $\frac{1}{x-2} = \frac{2}{x^2-2x}$ ☐

1.2 $\frac{2x-30}{x+3} - 2 = 6 \cdot \frac{x+3}{x^2-9}$ ☒

1.3 $\frac{x^2+3x+4}{x^2+3x+2} = 1$ ☐

1.4 $\frac{x+1}{x-1} - \frac{3}{x-1} = 1$ ☒

2.0 Bestimmen Sie die Definitionsmenge und lösen Sie folgende Gleichungen durch Überkreuzmultiplikation.

2.1 $\frac{x}{x+2} = \frac{2x}{2x+9}$ ☐

2.2 $\frac{x+6}{x} = \frac{x+4}{x+1}$ ☐

2.3 $\frac{x}{x-7} = \frac{x+5}{x-3}$ ☐

2.4 $\frac{3x+2}{3x-1} = \frac{6x}{6x-1}$ ☐

2.5 $\frac{2}{x-1} = x$ ☒

2.6 $\frac{2x+1}{3x-4} = \frac{x+6}{4x+1}$ ☒

Lösungen

1.1

$$D = \mathbb{R} \setminus \{0; 2\}$$

$$x^2 - 2x = 2x - 4 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow x = 2 \Rightarrow IL = \{ \}$$

1.2

$$D = \mathbb{R} \setminus \{-3; 3\}$$

$$\frac{2x - 30 - 2x - 6}{x + 3} = \frac{6x + 18}{x^2 - 9} \Rightarrow \frac{-36}{x + 3} = \frac{6x + 18}{x^2 - 9} \Rightarrow -36x^2 + 324 = 6x^2 + 36x + 54$$

$$\Rightarrow -42x^2 - 36x + 270 = 0 \Rightarrow x_1 = -3 \quad x_2 = \frac{15}{7} \Rightarrow IL = \left\{ \frac{15}{7} \right\}$$

1.3

$$D = \mathbb{R} \setminus \{-2; -1\} \quad x^2 + 3x + 2 = (x + 1) \cdot (x + 2)$$

$$x^2 + 3x + 4 = x^2 + 3x + 2 \Rightarrow 4 = 2 \text{ (f)} \Rightarrow IL = \{ \}$$

1.4

$$D = \mathbb{R} \setminus \{1\}$$

$$\frac{x + 1 - 3}{x - 1} = 1 \Rightarrow x - 2 = x - 1 \Rightarrow -2 = -1 \text{ (f)} \Rightarrow IL = \{ \}$$

2.1

$$D = \mathbb{R} \setminus \{-4, 5; -2\}$$

$$2x^2 + 9x = 2x^2 + 4x \Rightarrow 5x = 0 \Rightarrow x = 0 \Rightarrow IL = \{0\}$$

2.2

$$D = \mathbb{R} \setminus \{-1; 0\}$$

$$x^2 + 7x + 6 = x^2 + 4x \Rightarrow 3x = -6 \Rightarrow x = -2 \Rightarrow IL = \{-2\}$$

2.3

$$D = \mathbb{R} \setminus \{3; 7\}$$

$$x^2 - 3x = x^2 - 2x - 35 \Rightarrow -x = -35 \Rightarrow x = 35 \Rightarrow IL = \{35\}$$

2.4

$$D = \mathbb{R} \setminus \left\{ \frac{1}{6}; \frac{1}{3} \right\}$$

$$18x^2 + 9x - 2 = 18x^2 - 6x \Rightarrow 15x = 2 \Rightarrow x = \frac{2}{15} \Rightarrow IL = \left\{ \frac{2}{15} \right\}$$

2.5

$$D = \mathbb{R} \setminus \{1\}$$

$$2 = x^2 - x \Rightarrow x^2 - x - 2 = 0 \Rightarrow x_1 = -1 \quad x_2 = 2 \Rightarrow IL = \{-1; 2\}$$

2.6

$$D = \mathbb{R} \setminus \left\{ -\frac{1}{4}; \frac{4}{3} \right\}$$

$$8x^2 + 6x + 1 = 3x^2 + 14x - 24 \Rightarrow 5x^2 - 8x + 25 = 0 \Rightarrow IL = \{ \}$$